
UNIT 2 FRICTION

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2.1 INTRODUCTION

When a body moves or tends to move on another body, a force appears between the surfaces. This force is called force of friction and it acts opposite to the direction of motion. Its line of action is tangential to the contacting surfaces. The magnitude of this force depends on the roughness of surfaces.

In engineering applications friction is desirable and undesirable. We can walk on the ground because of friction. Friction is useful in power transmission by belts. It is useful in appliances like brakes, bolts, screw jack, etc. It is undesirable in bearing and moving machine parts where it results in loss of energy and, thereby, reduces efficiency of the machine.

In this unit, you will study screw jack, clutches and different type of bearings.

Objectives

After studying this unit, you should be able to

- know application of friction force,
- know theory background of screw jack,
- know different type of bearings,
- analyse bearings, and
- know function of clutches.

2.2 TYPES OF FRICTION

There are two types of friction :

- (a) Friction in un-lubricated surfaces or dry surfaces, and
- (b) Friction in lubricate surfaces.

The friction that exists when one dry surface slides over another dry surface is known as *dry friction* and the *friction*.

If between the two surfaces a thick layer of an oil or lubricant is introduced, a film of such lubrication is formed on both the surfaces. When a surface moves on the other, in effect, it is one layer of oil moving on the other and there is no direct contact between the surfaces. The friction is greatly reduced and is known as *film friction*.

2.3 LAWS OF DRY FRICTION

The laws of dry friction, are based on experimental evidences, and as such they are empirical in nature :

- (a) The friction force is directly proportional to the normal reaction between the surfaces.
- (b) The frictional force opposes the motion or its tendency to the motion.
- (c) The frictional force depends upon the nature of the surfaces in contact.
- (d) The frictional force is independent of the area and the shape of the contacting surfaces.
- (e) For moderate speeds, frictional force is independent of the relative velocities of the bodies in contact.

SAQ 1

On which factors friction force depends?

2.4 STATIC AND KINETIC FRICTION

Suppose a block of weight W rests on a plane surface as shown in Figure 2.1. The surface offers a normal reaction R_N equal to the weight W . Suppose, now, a pull P_1 is applied to the block such that it actually does not move but instead tends to move. This will be opposed by frictional force F_1 , equal to P_1 . The resultant reaction will be R_1

inclined at an angle ϕ_1 with the normal reaction. If the pull is increased to P_2 , the frictional force will increase to F_2 and the resultant reaction will increase to R_2 inclined at an angle of ϕ_2 . Thus, with increase of the pull or attractive force, the frictional force; the resultant reaction and its inclination will increase.

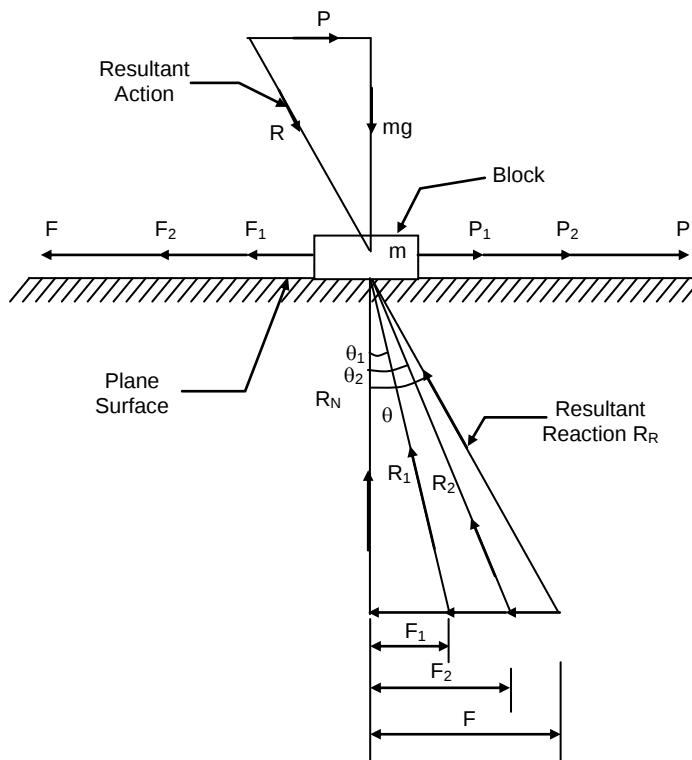


Figure 2.1 : Static and Kinetic Friction

Figure 2.1 shows a block of mass m resting on a plane surface with application of force P_1 , P_2 and P_3 at which the body impends sliding, the self adjusting frictional forces will increase from F_1 , to F_2 and finally F_3 when the body tends to move. Thus, in the limiting condition, the resultant active force will be R and reactive force R_R .

The frictional resistance offered so long as the body does not move, is known as *static friction*. F_1 and F_2 are the static frictional forces. It may be noted that the direction of the resultant reaction R_R is such that it opposes the motion.

The ultimate value of static friction (F) when the body just tends to move is called *limiting friction* or *maximum static friction* or *friction of impending slide*. The condition, when all the forces are just in equilibrium and the body has a tendency to move, is called *limiting equilibrium position*. When a body moves relative to another body, the resisting force between them is called *kinetic* or *sliding friction*. It has been experimentally found that the kinetic friction is less than the maximum static friction.

SAQ 2

What is the value of limiting friction?

2.5 COEFFICIENT OF FRICTION

The ratio between the maximum static frictional force and the normal reaction R_N remains constant which is known as *coefficient of static friction* denoted by Greek letter μ .

$$\therefore \text{Coefficient of friction} = \frac{\text{Maximum static frictional force}}{\text{Normal reaction}}$$

or
$$\mu = \frac{F}{R_N} \quad \dots (2.1)$$

The maximum angle ϕ which the resultant reaction R_N makes with the normal reaction R_N is known as *angle of friction*. It is denoted by ϕ .

From Figure 2.1
$$\tan \phi = \frac{F}{R_N} = \mu$$

$$\Rightarrow \phi = \tan^{-1} \mu \quad \dots (2.2)$$

The coefficient of friction is different for different substances and even varies for different conditions of the same two surfaces.

Approximate values of static coefficient of friction for dry (un-lubricated) and greasy lubricated surfaces are given as below :

Materials	Dry	Greasy
Hard steel on hard steel	0.42	0.029-0.108
Mild steel on mild steel	0.57	0.09-0.19
Wooden wood	0.2-0.5	0.133
Mild steel on cast iron	0.24	0.09-0.116
Wood on metal	0.2-0.6	—
Glass on glass	0.4	—
Metal on stone	0.3-0.7	—
Metal on leather	0.3-0.6	—
Wood on leather	0.2-0.5	—
Earth on earth	0.1-1	—
Cast iron on cast iron	0.3-3.4	0.065-0.070
Rubber on concrete	0.65-0.85	—
Rubber on ice	0.05-0.2	—

2.6 ANGLE OF REPOSE

Consider a mass m resting on an inclined plane. If the angle of inclination is slowly increased, a stage will come when the block of mass m will tend to slide down (Figure 2.2). This angle of the plane with horizontal plane is known as *angle of repose*. For satisfying the conditions of equilibrium all the forces are resolved parallel to the plane and perpendicular to it.

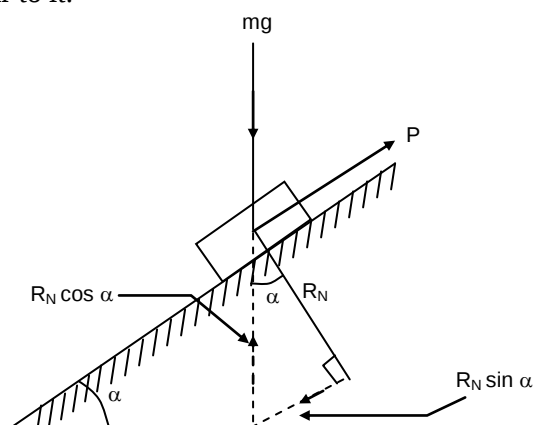


Figure 2.2 : Angle of Repose

$$P = R_N \sin \alpha$$

$$mg = R_N \cos \alpha$$

$$\frac{P}{mg} = \tan \alpha$$

But $\frac{P}{mg} = \frac{F}{R_N} = \mu = \tan \phi$

$$\Rightarrow \quad \tan \alpha = \tan \phi$$

Angle of repose α = angle of friction ϕ .

2.7 LEAST FORCE REQUIRED TO DRAG A BODY ON A ROUGH HORIZONTAL PLANE

Suppose a block, of mass m , is placed on a horizontal rough surface as shown in Figure 2.3 and a tractive force P is applied at an angle θ with the horizontal such that the block just tends to move.

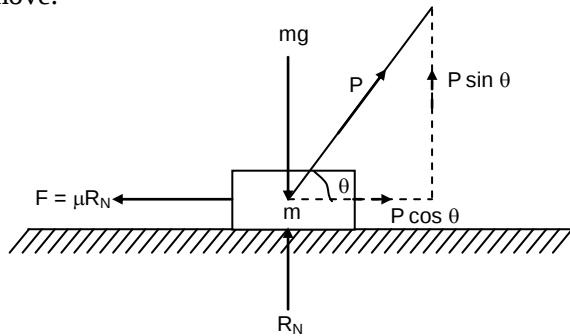


Figure 2.3

For satisfying the equilibrium conditions the forces are resolved vertically and horizontally.

For $\Sigma V = 0$

$$mg - P \sin \theta = R_N$$

or $R_N = (mg - P \sin \theta) \quad \dots (2.3)$

For $\Sigma H = 0$

$$P \cos \theta = F = \mu R_N = \mu (mg - P \sin \theta)$$

[substituting the value of R_N from Eq. (2.3)]

$$P \cos \theta = \frac{\sin \phi}{\cos \phi} (mg - P \sin \theta)$$

$$P \cos \theta \cos \phi = mg \sin \phi - P \sin \theta \sin \phi$$

$$P \cos \theta \cos \phi + P \sin \theta \sin \phi = mg \sin \phi$$

$$P (\cos \theta \cos \phi + \sin \theta \sin \phi) = mg \sin \phi$$

$$P \cos (\theta - \phi) = mg \sin \phi$$

$$P = \frac{mg \sin \phi}{\cos (\theta - \phi)} \quad \dots (2.4)$$

For P to be least, the denominator $\cos (\theta - \phi)$ must be maximum and it will be so if

$$\cos (\theta - \phi) = 1 \quad \text{or} \quad \theta - \phi = 0$$

$$\Rightarrow \theta = \phi \text{ for least value of } P$$

$$\Rightarrow P_{\text{least}} = mg \sin \phi \quad \dots (2.5)$$

Hence, the force P will be the least if angle of its inclination with the horizontal : θ is equal to the angle of friction ϕ .

2.8 HORIZONTAL FORCE REQUIRED TO MOVE THE BODY

Up the Inclined Plane

The force P has been applied to move the body up the plane. Resolving all the forces parallel and perpendicular to the plane and writing equations, we get

$$P \cos \alpha = \mu R_N + mg \sin \alpha$$

$$P \sin \alpha = R_N - mg \cos \alpha$$

$$\therefore R_N = P \sin \alpha + mg \cos \alpha$$

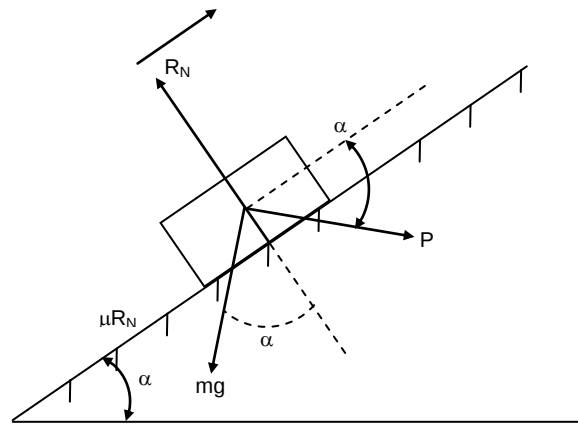


Figure 2.4

Substituting the value of R_N , we get

$$P \cos \alpha = \mu (P \sin \alpha + mg \cos \alpha) + mg \sin \alpha$$

Assuming $\mu = \tan \phi$

where ϕ is angle of friction.

$$P \cos \alpha = \tan \phi (P \sin \alpha + mg \cos \alpha) + mg \sin \alpha$$

$$\text{or } P (\cos \alpha - \tan \phi \sin \alpha) = mg (\tan \phi \cos \alpha + \sin \alpha)$$

$$\text{or } P = mg \frac{\left(\frac{\sin \phi}{\cos \phi} \cos \alpha + \sin \alpha \right)}{\left(\cos \alpha - \frac{\sin \phi}{\cos \phi} \sin \alpha \right)}$$

$$= mg \frac{(\sin \phi \cos \alpha + \sin \alpha \cos \phi)}{(\cos \alpha \cos \phi - \sin \phi \sin \alpha)}$$

$$= mg \frac{\sin (\alpha + \phi)}{\cos (\alpha + \phi)} = mg \tan (\alpha + \phi) \quad \dots (2.6)$$

Down the Plane

In this case body is moving down the plane due to the application of force P . Resolving the forces parallel and perpendicular to the plane and writing the equations, we get

$$P \cos \alpha = \mu R_N - mg \sin \alpha$$

$$P \sin \alpha = mg \cos \alpha - R_N$$

$$\therefore R_N = mg \cos \alpha - P \sin \alpha$$

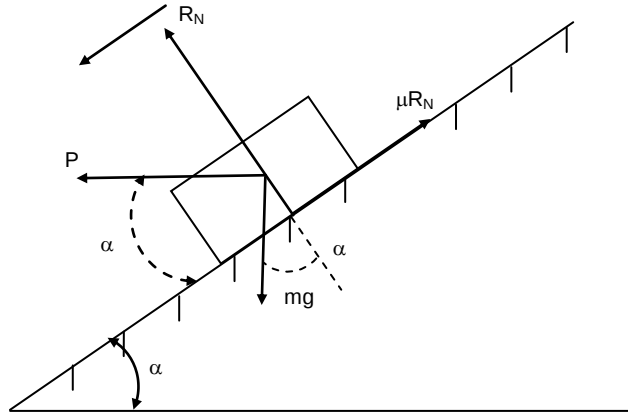


Figure 2.5

Substituting for R_N , we get

$$P \cos \alpha = \mu (mg \cos \alpha - P \sin \alpha) - mg \sin \alpha$$

Since $\mu = \tan \phi$

$$\therefore P \cos \alpha = \tan \phi (mg \cos \alpha - P \sin \alpha) - mg \sin \alpha$$

$$\text{or } P (\cos \alpha + \tan \phi \sin \alpha) = mg (\tan \phi \cos \alpha - \sin \alpha)$$

$$\begin{aligned} \text{or } P &= mg \frac{\left(\frac{\sin \phi}{\cos \phi} \cos \alpha - \sin \alpha \right)}{\left(\cos \alpha + \frac{\sin \phi}{\cos \phi} \sin \alpha \right)} \\ &= mg \frac{(\sin \phi \cos \alpha - \cos \phi \sin \alpha)}{(\cos \alpha \cos \phi + \sin \phi \sin \alpha)} \end{aligned}$$

$$\text{or } P = mg \frac{\sin (\phi - \alpha)}{\cos (\phi - \alpha)} = mg \tan (\phi - \alpha) \quad \dots (2.7)$$

This means for force P to be applied $\alpha < \phi$.

For $\phi < \alpha$ body will move without applying force P .

2.9 SCREW AND NUT FRICTION

Now consider screw and nut assembly. Both of them have threads in the form of helix. Screw has external thread and nut has internal thread. The pitch of threads (p) for both is same. When nut is rotated by one turn the screw traverses linear distance equal to the pitch (p). The nut at the same time traverses by one turn. These distances so traveled have been represented in Figure 2.6(a). Let mean diameter of the screw be ' dm '.

$$\text{Therefore, } \tan \alpha = \frac{p}{\pi dm}$$

The theory of the inclined plane discussed in the preceding article can be applied for determining the effort to be applied.

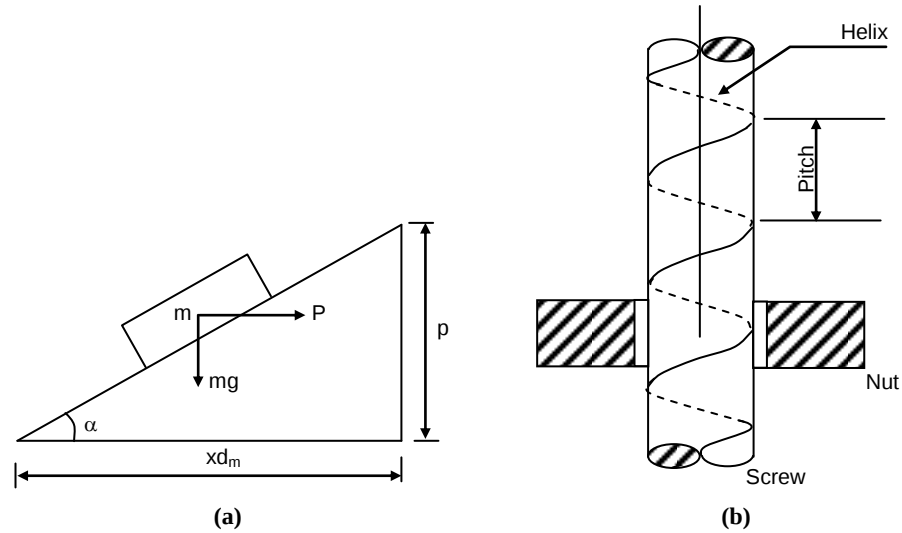


Figure 2.6 : Screw and Nut Friction

For Upward Motion

Effort $P_0 = mg \tan \alpha$ without friction [$\phi = 0$].

Effort $P = mg \tan (\alpha + \phi)$ with friction.

$$\therefore \text{Efficiency } e_{up} = \frac{\tan \alpha}{\tan (\alpha + \phi)}$$

For Downward Motion

$$P = mg \tan (\phi - \alpha)$$

2.10 SELF-LOCKING SCREWS

If $\alpha > \phi$, the mass placed on screw will start moving downward by its own weight and force P shall have to be applied to hold it. To guard against this undesirable effect, the screws angle α is always kept less than angle ϕ . Such a screw is known as *self-locking screw*.

2.11 CONDITION FOR MAXIMUM EFFICIENCY

$$\text{Efficiency of screw : } e_{up} = \frac{\tan \alpha}{\tan (\alpha + \phi)}$$

For determining condition of maximum efficiency,

$$\frac{d e_{up}}{d \alpha} = 0 = \frac{\sec^2 \alpha \tan (\alpha + \phi) - \sec^2 (\alpha + \phi) \tan \alpha}{\tan^2 (\alpha + \phi)} = 0$$

$$\sec^2 \alpha \tan (\alpha + \phi) = \sec^2 (\alpha + \phi) \tan \alpha$$

$$\sin (\alpha + \phi) \cos (\alpha + \phi) = \sin \alpha \cos \alpha$$

$$\sin 2 (\alpha + \phi) = \sin 2 \alpha$$

$$2 (\alpha + \phi) = (\pi - 2 \alpha)$$

$$\text{or } \alpha = \frac{\pi}{4} - \frac{\phi}{2}$$

Substituting the value of α in equation of efficiency.

Friction

$$e_{\max} = \frac{\tan \left(45^\circ - \frac{\phi}{2} \right)}{\tan \left(45^\circ + \frac{\phi}{2} \right)} = \frac{\frac{1 - \tan \frac{\phi}{2}}{1 + \tan \frac{\phi}{2}}}{\frac{1 + \tan \frac{\phi}{2}}{1 - \tan \frac{\phi}{2}}} = \frac{\left(1 - \tan \frac{\phi}{2} \right)^2}{\left(1 + \tan \frac{\phi}{2} \right)^2} = \frac{\left(\cos \frac{\phi}{2} - \sin \frac{\phi}{2} \right)^2}{\left(\cos \frac{\phi}{2} + \sin \frac{\phi}{2} \right)^2}$$

$$= \frac{1 - 2 \sin \frac{\phi}{2} \cos \frac{\phi}{2}}{1 + 2 \sin \frac{\phi}{2} \cos \frac{\phi}{2}} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$e_{\max} = \frac{1 - \sin \phi}{1 + \sin \phi}$$

SAQ 3

How self locking is provided in screw jack?

2.12 SCREW JACK

With Square Threads

A screw jack with its spindle, having square threads, is shown in Figure 2.7. The theory discussed till now is directly applicable to this case.

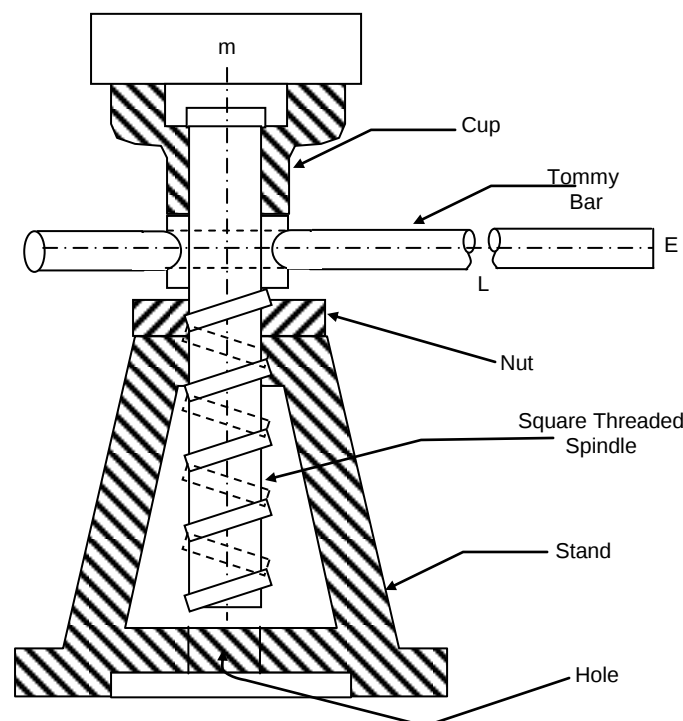


Figure 2.7 : Screw Jack

Let m = Mass on the jack,

P = Force applied at the screw tangentially in a horizontal plane,

P_e = Horizontal force applied tangentially at the end E of a tommy bar in a horizontal plane, and

L = Horizontal distance between central axis of the screw and the end E of the bar as shown.

In this screw jack nut is stationary and the screw is rotated with the help of tommy bar.

$$P_e \times L = P \times r$$

$$P_e = \frac{P \times r}{L}$$

$$P = mg \tan (\alpha + \phi) = \frac{mg (\tan \alpha + \tan \phi)}{1 - \tan \alpha \tan \phi}$$

But $\tan \alpha = \frac{p}{\pi d_m}$ and $\tan \phi = \mu$

Substituting for $\tan \alpha$ and $\tan \phi$

$$\Rightarrow P = \frac{mg \left(\frac{p}{\pi d_m} + \mu \right)}{1 - \frac{p}{\pi d_m} \times \mu} = \frac{mg (p + \mu \pi d_m)}{(\pi d_m - p \mu)}$$

Hence, $P = \frac{mg (p + \mu \pi d_m)}{(\pi d_m - p \mu)}$

$$P_e = \frac{\frac{mg \times r}{L} (p + \mu \pi d_m)}{(\pi d_m - p \mu)}$$

Mechanical advantage of the jack with tommy bar

$$\begin{aligned} &= \frac{\text{Load lifted}}{\text{Force applied}} \\ &= \frac{mg}{P_e} = \frac{L}{r} \times \frac{(\pi d_m - p \mu)}{(p + \mu \pi d_m)} \end{aligned}$$

Velocity ratio with tommy bar :

$$\begin{aligned} &= \frac{\text{Distance covered by } P_e}{\text{Distance covered by load in one revolution}} \\ &= \frac{2\pi L}{2\pi r_m \times \tan \alpha} = \frac{L}{r_m \tan \alpha} \end{aligned}$$

$$\text{V. R.} = \frac{2\pi L}{p}$$

With V-threads

The square threads, by their nature, take the axial load mg perpendicular to them where as in V-threads, the axial load does not act perpendicular to the surface of the threads as shown in Figure 2.8. The normal reaction R_N between the threads and the screw should be such that its axial component is equal and opposite to the axial load mg .

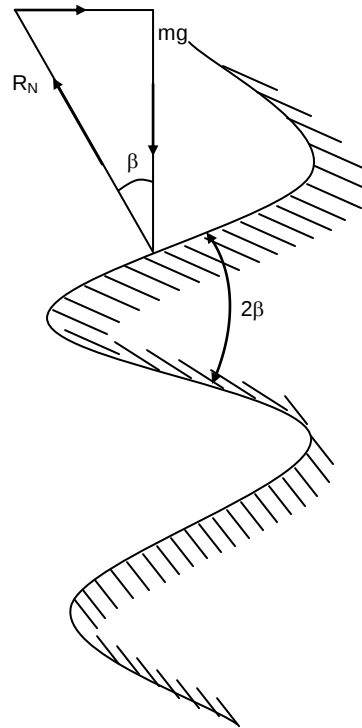


Figure 2.8 : V-threads

Let 2β be the angle included between the two sides of the thread.

$$R_N \cos \beta = mg$$

$$R_N = \frac{mg}{\cos \beta}$$

Frictional force which acts tangential to the surface of the threads = μR_N

$$= \frac{\mu \times mg}{\cos \beta} = \mu_1 \times mg$$

where μ_1 may be regarded as virtual coefficient of friction :

$$\Rightarrow \mu_1 = \frac{\mu}{\cos \beta}$$

While treating V-threads for finding out effort F or 'e', etc. μ may be substituting by μ_1 in all the relevant equations meant for the square threads.

It may be observed that force required to lift a given load with V-threads will be more than that with square threads.

Screw threads are also used for transmission of power such as in lathes (lead screw), milling machines, etc. The square threads will transmit power without any side thrust but is difficult to cut. The Acme threads, though not as efficient as the square threads are easier to cut.

Example 2.1

Outside diameter of a square threaded spindle of a screw jack is 40 mm. The screw pitch is 10 mm. If the coefficient of friction between the screw and the nut is 0.15, neglecting friction between the nut and the collar, determine :

- Force required to be applied at the end of tommy bar 1 m in length to raise a load of 20 kN.
- Efficiency of the screw.

Solution

Outside diameter of the screw : $D = 40 \text{ mm}$

Inside diameter of the screw :

$$d = 40 \text{ mm} - 10 = 30 \text{ mm}$$

Mean diameter of the screw :

$$d_m = \frac{40 + 30}{2} = 35 \text{ mm}$$

The force required for raising the load.

$$(a) \quad P = mg \frac{d_m}{2} \tan (\alpha + \phi) = \frac{mg (\tan \alpha + \tan \phi)}{1 - \tan \alpha \tan \phi} \times \frac{d_m}{2L}$$

$$\text{But } \tan \phi = \mu = 0.15$$

$$\tan \alpha = \frac{p}{\pi d_m} = \frac{10 \times 7}{22 \times 35} = 0.091$$

Substituting the values,

$$P = 20 \times \frac{(0.091 + 0.15)}{1 - 0.091 \times 0.15} \times \frac{30}{2 \times 1000} = 73 \text{ N}$$

We know that,

$$P_0 = mg \tan \alpha \frac{30}{2 \times 1000} = 20 \times 0.091 \times 15 = 27.3$$

$$(b) \quad \text{Efficiency } e_{up} = \frac{P_0}{P} = \frac{27.3}{73} \times 100 = 37.4\%$$

2.13 PIVOT AND COLLAR FRICTION

The shafts of ships, steam and water turbines are subjected to *axial thrust*. In order to take up the axial thrust, they are provided with one or more bearing surfaces at right angle to the axis of the shaft. A bearing surface provided at the end of a shaft is known as a *pivot* and that provided at any place along with the length of the shaft with bearing surface of revolution is known as a *collar*. Pivots are of two forms : flat and conical. The bearing surface provided at the foot of a vertical shaft is called *foot step bearing*.

Due to the axial thrust which is conveyed to the bearings by the rotating shaft, rubbing takes place between the contacting surfaces. This produces friction as well as wearing of the bearing. Thus, power is lost in over-coming the friction, which is ultimately to be determined in this unit.

Obviously, the rate of wearing depends upon the intensity of thrust (pressure) and relative velocity of rotation. Since velocity is proportional to the radius, therefore,

$$\therefore \quad \text{Rate of wear} \propto pr.$$

Assumptions Taken

- (a) Firstly, the intensity of pressure is uniform over the bearing surface. This assumption only holds good with newly fitted bearings where fit between the two contacting surfaces is assumed to be perfect. After the shaft has run for quite sometime the pressure distribution will not remain uniform due to varying wear at different radii.

- (b) *Secondly, the rate of wear is uniform.* As given previously, the rate of wear is proportional to pr which means that the pressure will go on increasing radially inward and at the centre where $r = 0$, the pressure will be infinite which is not true in practical sense. *However, this assumption of uniform wear gives better practical results when bearing has become older.*

The various types of bearings mentioned above will be dealt with separately for each assumption.

2.13.1 Flat Pivot

A flat pivot is shown in Figure 2.9.

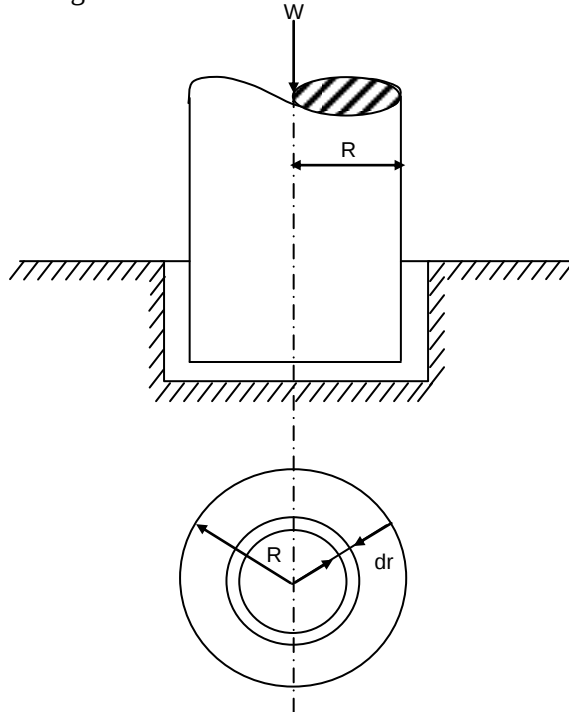


Figure 2.9 : Flat Pivot

Let W = Axial thrust or load on the bearing,

R = External radius of the pivot,

p = Intensity of pressure, and

μ = Coefficient of friction between the contacting surfaces.

Consider an elementary ring of the bearing surfaces, at a radius r and of thickness dr as shown in Figure 2.9.

Axial load on the ring

$$dW = p \times 2\pi r \times dr$$

$$\text{Total load } W = \int_0^R p \times 2\pi r \times dr \quad \dots (2.8)$$

Frictional force on the ring

$$dF = \mu \times dW = \mu \times p \times 2\pi r \times dr$$

Frictional moment about the axis of rotation

$$dM = dF \times r = \mu \times p \times 2\pi r^2 \times dr$$

Total frictional moment

$$M = \int_0^R dM = \int_0^R \mu \times p \times 2\pi r^2 \times dr \quad \dots (2.9)$$

Uniform Pressure

If the intensity of pressure p is assumed to be uniform and hence constant.

From Eq. (2.8)

$$W = p \times 2\pi \int_0^R r \times dr = p \times 2\pi \left[\frac{r^2}{2} \right]_0^R = p \times 2\pi \times \frac{R^2}{2}$$

$$\Rightarrow \quad \mathbf{W = p \times \pi R^2}$$

And from Eq. (2.9)

$$M = \mu \times p \times 2\pi \int_0^R r^2 dr$$

$$M = \mu p \times 2\pi \times \left[\frac{r^3}{3} \right]_0^R = \frac{2}{3} \mu \times p \times \pi \times R^3$$

But $p \pi R^2 = W$

$$\Rightarrow \quad \mathbf{M = \frac{2}{3} \mu WR = \mu W \times \frac{2}{3} R}$$

The friction force μW can be considered to be acting at a radius of $\frac{2}{3} R$.

Uniform Rate of Wear

By Eq. (2.8),

$$W = \int_0^R p \times 2\pi r \times dr$$

As the rate of wear is taken as constant and proportional to $pr = a$ constant say c .
Substituting for $pr = c$ in the above equation.

$$W = \int_0^R 2\pi \times c \times dr = 2\pi R \times c$$

$$\Rightarrow \quad c = \frac{W}{2\pi R} \quad \dots (2.10)$$

By Eq. (2.9), total frictional moment

$$\begin{aligned} M &= \int_0^R \mu \times p \times 2\pi r^2 \times dr = \int_0^R \mu \times 2\pi \times c \times r \times dr \\ &= \mu \times 2\pi \times c \times \frac{R^2}{2} = \mu \times 2\pi \times \frac{W}{2\pi R} \times \frac{R^2}{2} \end{aligned}$$

$$\therefore \quad \mathbf{M = \mu W \times \frac{R}{2}} \quad \dots (2.11)$$

Thus, the frictional force : μW acts at a distance $\frac{R}{2}$ from the axis.

2.13.2 Conical Pivot

A truncated conical pivot is shown in Figure 2.10(a).

Let 2α = The cone angle,

W = The axial load/thrust,

R_1 = The outer radius of the cone,

R_2 = The inner radius of the cone, and

p = Intensity of pressure which will act normal to the inclined surface of the cone as shown in Figure 2.10(b).

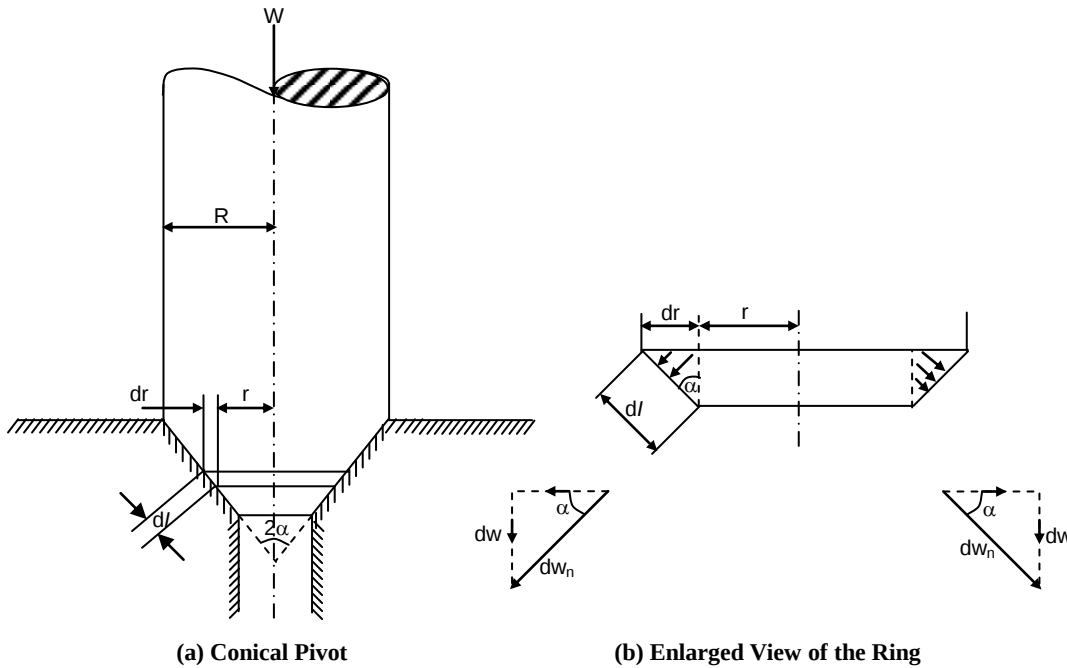


Figure 2.10 : Truncated Conical Pivot

Consider an elementary ring of the cone, of radius r thickness dr and of sloping length dl as shown. Enlarged view of the ring is shown in Figure 2.10(b).

Normal load on the ring

$$dW_n = p \times 2\pi r \times dl = p \times 2\pi r \times \frac{dr}{\sin \alpha}$$

Axial load on the ring

$$\begin{aligned} dW &= dW_n \times \sin \alpha \\ &= p \times 2\pi r \times \frac{dr}{\sin \alpha} \times \sin \alpha = p \times 2\pi r \times dr \end{aligned}$$

Total axial load on the bearing

$$W = \int_{R_2}^{R_1} p \times 2\pi r \times dr \quad \dots (2.12)$$

Frictional force on the elementary ring

$$dF = \mu dW_n = \mu \times p \times 2\pi r \times dl = \mu \times p \times 2\pi r \times \frac{dr}{\sin \alpha}$$

Moment of the frictional force about the axis of rotation

$$dM = \mu \times p \times 2\pi r^2 \times \frac{dr}{\sin \alpha}$$

$$M = \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times \frac{dr}{\sin \alpha} \quad \dots (2.13)$$

Uniform Pressure

Uniform pressure, p is constant

From Eq. (2.12),

$$W = p \times 2\pi \int_{R_2}^{R_1} r \, dr = p \times 2\pi \left[\frac{r^2}{2} \right]_{R_2}^{R_1}$$

$$\text{or} \quad W = p \times \frac{2\pi}{2} (R_1^2 - R_2^2) = p \times \pi (R_1^2 - R_2^2) \quad \dots (2.14)$$

And from Eq. (2.13),

$$\therefore M = \frac{\mu \times p \times 2\pi}{\sin \alpha} \int_{R_2}^{R_1} r^2 \, dr = \frac{\mu \times p \times 2\pi}{\sin \alpha} \times \frac{1}{3} (R_1^3 - R_2^3)$$

But from Eq. (2.14)

$$W = p \times \pi (R_1^2 - R_2^2)$$

$$\therefore p = \frac{W}{(\pi R_1^2 - R_2^2)}$$

Substituting for p above,

$$\Rightarrow M = \frac{\mu W}{\sin \alpha} \times \frac{2}{3} \frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}$$

Thus, frictional force $\frac{\mu W}{\sin \alpha}$ acts at a radius of $\frac{2}{3} \times \frac{(R_1^3 - R_2^3)}{(R_1^2 - R_2^2)}$

For a full conical pivot shown in Figure 2.11, $R_2 = 0$

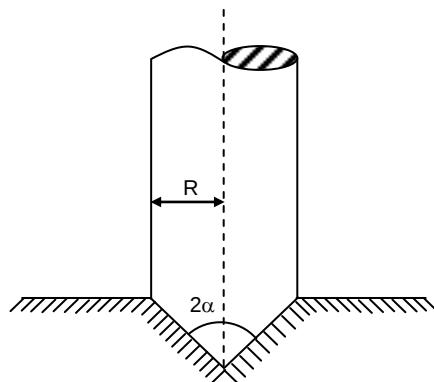


Figure 2.11 : Full Conical Pivot

$$r = 0$$

$$\Rightarrow W = p \pi R_1^2$$

$$\text{and} \quad M = \frac{\mu W}{\sin \alpha} \times \frac{2}{3} R_1$$

In this case the frictional force $\left(\frac{\mu W}{\sin \alpha} \right)$ acts at a radius $\frac{2}{3}$ from the axis.

From Eq. (2.12),

$$W = \int_r^R p \times 2\pi r \times dr$$

Since the rate of wear is uniform.

Therefore, $pr = \text{constant } c$

Substituting for pr

$$W = \int_{R_2}^{R_1} 2\pi c \times dr = 2\pi c \int_{R_2}^{R_1} dr = 2\pi c \times (R_1 - R_2)$$

$$\Rightarrow c = \frac{W}{2\pi (R_1 - R_2)} \quad \dots (2.15)$$

From Eq. (2.13), frictional moment

$$\begin{aligned} M &= \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times \frac{dr}{\sin \alpha} \\ &= \int_{R_2}^{R_1} \mu \times 2\pi r \times c \times \frac{dr}{\sin \alpha} \quad [\text{since } pr = c] \\ &= \frac{\mu \times 2\pi c}{\sin \alpha} \int_{R_2}^{R_1} r dr \\ &= \frac{\mu \times 2\pi c}{\sin \alpha} \frac{(R_1^2 - R_2^2)}{2} = \frac{\mu \times \pi c}{\sin \alpha} (R_1^2 - R_2^2) \end{aligned}$$

$$\text{But } c = \frac{W}{2\pi (R_1 - R_2)} \quad \dots (2.16)$$

$$\Rightarrow M = \frac{\mu W}{\sin \alpha} \times \frac{(R_1 + R_2)}{2}$$

[Thus, the frictional force $\frac{\mu W}{\sin \alpha}$ acts at a radius $= \frac{(R_1 + R_2)}{2}$]

For the full conical pivot (Figure 2.10), $R_2 = 0$

$$M = \frac{\mu W}{\sin \alpha} \times \frac{R_1}{2}$$

Thus, the friction force $\frac{\mu W}{\sin \alpha}$ acts at a radius $\frac{R_1}{2}$.

2.13.3 Collar Bearing

A collar bearing which is provided on to a shaft, is shown in Figure 2.12.

Let $W =$ The axial load/thrust,

$R_1 =$ External radius of the collar, and

$R_2 =$ Internal radius of the collar.

Consider an elementary ring of the collar surface, of radius r and of thickness dr as shown in Figure 2.12.

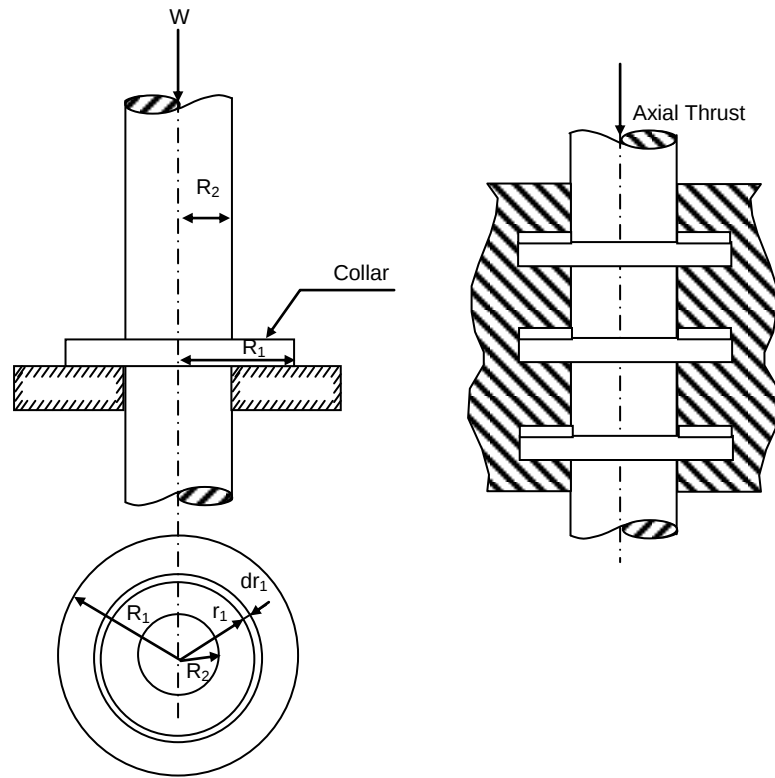


Figure 2.12 : Collar Bearing

Axial load on the ring

$$dW = p \times 2\pi r \times dr$$

Total axial load

$$W = \int_{R_2}^{R_1} p \times 2\pi r \times dr \quad \dots (2.17)$$

Frictional force on the ring

$$dF = \mu \times p \times 2\pi r \times dr$$

Frictional moment of the ring

$$dM = \mu \times p \times 2\pi r^2 \times dr$$

Total frictional moment

$$M = \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times dr \quad \dots (2.18)$$

Uniform Pressure

Uniform pressure, p is constant

From Eq. (2.17),

$$W = p \times 2\pi \int_{R_2}^{R_1} r \times dr = p \times 2\pi \frac{(R_1^2 - R_2^2)}{2}$$

$$\Rightarrow W = p \times \pi (R_1^2 - R_2^2) \quad \dots (2.19)$$

From Eq. (2.18),

$$M = \mu \times p \times 2\pi \int_{R_2}^{R_1} r^2 \times dr = \mu \times p \times 2\pi \frac{(R_1^3 - R_2^3)}{3}$$

Substituting for p from above

$$\Rightarrow M = \mu W \times \frac{2 (R_1^3 - R_2^3)}{3 (R_1^2 - R_2^2)} \quad \dots (2.20)$$

Uniform Wear

From Eq. (2.17),

Since the rate of wear is uniform

$$\therefore pr = \text{constant } c$$

Substituting for pr above,

$$W = \int_r^R c \times 2\pi dr = c \times 2\pi \int_{R_2}^{R_1} dr = c \times 2\pi \times (R_1 - R_2)$$

$$c = \frac{W}{2\pi (R_1 - R_2)} \quad \dots (2.21)$$

From Eq. (2.18),

$$M = \int_{R_2}^{R_1} \mu \times p \times 2\pi r^2 \times dr = \mu \times 2\pi \times c \int_{R_2}^{R_1} r \times dr$$

$$= \mu \times 2\pi \times c \frac{(R_1 - R_2)}{2}$$

Substituting for c from Eq. (2.21),

$$\Rightarrow M = \mu W \times \frac{(R_1 + R_2)}{2} \quad \dots (2.22)$$

There is a limit to the bearing pressure on a single collar and it is about 40 N/cm^2 . Where the axial load is more and pressure on each collar is not to be allowed to exceed beyond the designed limit, then more collars are provided as shown in Figure 2.12.

$$\text{Number of collars : } n = \frac{\text{Total axial load}}{\text{Permissible axial load on each collar}}$$

It may be pointed out there is no change in the magnitude of frictional moments with more number of collars. The number of collars, as given above, only limit the maximum intensity of pressure in each collar.

Table 2.1 : Pivot and Collars Summary of Formulae

Sl. No.	Particular	Frictional Moments : M	
		Uniform Pressure	Uniform Wear
1.	Flat pivot	$\mu W \times \frac{2}{3} R$	$\mu W \times \frac{R}{2}$
2.	Conical pivot (a) Truncated	$\frac{\mu W}{\sin \alpha} \times \frac{2 (R_1^3 - R_2^3)}{3 (R_1^2 - R_2^2)}$	$\frac{\mu W}{\sin \alpha} \times \frac{(R_1 + R_2)}{2}$
	(b) Full conical	$\mu W \times \frac{2}{3} R$	$\mu W \times \frac{R}{2}$
3.	Collar	$\mu W \times \frac{2 (R_1^3 - R_2^3)}{3 (R_1^2 - R_2^2)}$	$\mu W \times \frac{(R_1 + R_2)}{2}$

SAQ 4

When collar or pivot bearing becomes older which assumption is more suitable and why?

2.14 CLUTCH

It is a mechanical device which is widely used in automobiles for the purpose of engaging driving and the driven shaft, at the will of the driver or the operator. The driving shaft is the engine crankshaft and the driven shaft is the gear-box driving shaft. This means that the clutch is situated between the engine flywheel mounted on the crankshaft and the gear box.

In automobile, gears are required to be changed for obtaining different speeds. It is possible only if the driving shaft of the gear box is stopped for a while without stopping the engine. These two objects are achieved with the help of a clutch.

Broadly speaking, a clutch consists of two members; one fixed to the crankshaft or the flywheel of the engine and the other mounted on a splined shaft, of the gear box so that this could be engaged or disengaged as the case may be with the member fixed to the engine crankshaft.

2.15 TYPES OF CLUTCHES

Clutches can be classified into two types as follows :

- (a) conical clutch, and
- (b) the plate or disc clutches can be of single plate or of multiple plates.

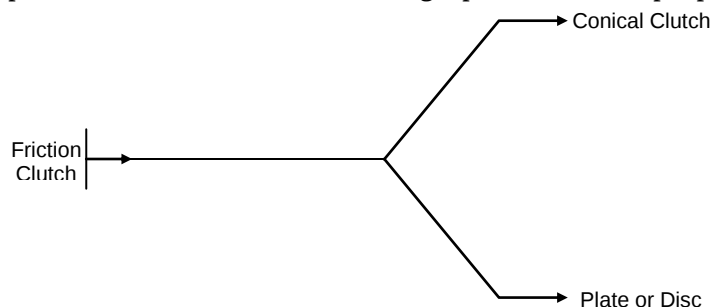


Figure 2.13 : The Single Cone Clutch

2.15.1 Conical Clutch

A conical clutch is shown in Figure 2.14. It consists of a cone *A* mounted on engine crankshaft. Cone *B* has internal splines in its boss which fit into the corresponding splines provided in the gear box shaft. Cone *B* could rotate the gear box shaft as well as may slide along with it. The outer surface of cone *B* is lined with friction material. In the normal or released position of the clutch pedal *P*, cone *B* fits into the inner conical surface of cone *A* and by means of the friction between the contacting surfaces, power is transmitted from crankshaft to the gear box shaft. When the clutch pedal is pressed, pivot *D* being the fulcrum provided in it, the collar *E* is pressed towards the right side, thus disengaging cone *B* from cone *A* and keeping the compression spring *S* compressed. On releasing the pedal, by the force of the spring, the cone *B* is thrust back to engage cone *A* for power transmission.

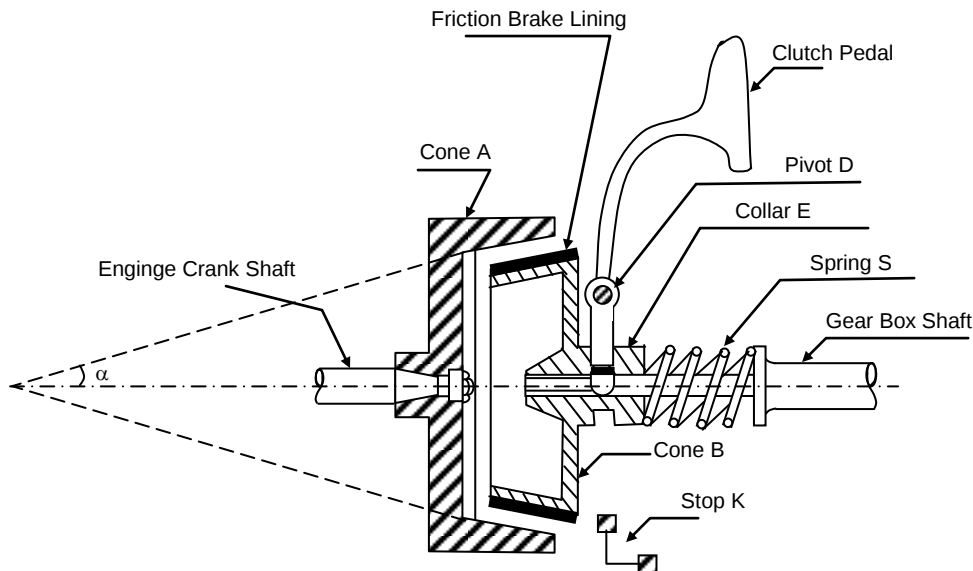


Figure 2.14 : Conical Clutch

For calculating frictional moments or torque transmitted on account of friction in clutches, unless otherwise specifically stated uniform rate of wear is assumed. For torque transmitted formulae of conical pivot can be used.

2.15.2 Single Plate Clutch

A single plate clutch is known as *single disc clutch*. It is shown in Figure 2.15. It has two sides which are driving and the driven side. The driving side comprises of the driving shaft or engine crankshaft A. A boss B is keyed to it to which flywheel C is bolted as shown. On the driven side, there is a driven shaft D. It carries a boss E which can freely slide axially along with the driven shaft through splines F. The clutch plate is mounted on the boss E. It is provided with rings of friction material – known as *friction linings*, on the both sides indicated H. One friction lining is pressed on the flywheel face and the other on the pressure plate I. A small spigot, bearing J, is provided in the end of the driving shaft for proper alignment.

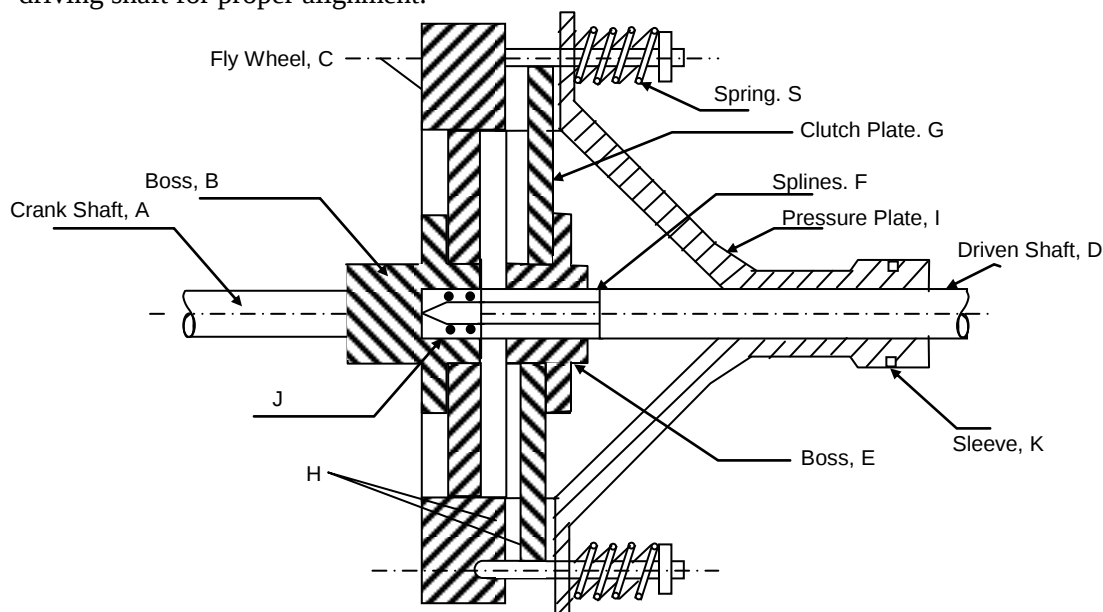


Figure 2.15 : Single Plate Clutch

The pressure plate provides axial thrust or pressure between clutch plate G and the flywheel C and the pressure plate I through the linings on its either side, by means of the springs, S.

The pressure plate remains engaged and as such clutch remains in operational position. Power from the driving shaft is transferred to the driven shaft from flywheel to the clutch

plate through the friction lining between them. From pressure plate the power is transmitted to clutch plate through friction linings. Both sides of the clutch plate are effective. When the clutch is to be disengaged the sleeve K is moved towards right hand side by means of clutch pedal mechanism (it is not shown in the figure). By doing this, there is no pressure between the pressure plate, flywheel and the clutch plate and no power is transmitted. In medium size and heavy vehicles, like truck, single plate clutch is used.

2.15.3 Multi Plate Clutch

As already explained in a plate clutch, the torque is transmitted by friction between one or more pairs of co-axial annular faces kept in contact by an axial thrust provided by springs. In a single plate clutch, both sides of the plate are effective so that it has two pairs of surfaces in contact or $n = 2$.

Obviously, in a single plate clutch limited amount of torque can be transmitted. When large amount of torque is to be transmitted, more pair of contact surfaces are needed and it is precisely what is obtained by a multi-plate clutch.

SAQ 5

- Clutches are used for which purpose?
- Where do we use single plate clutch. Name the vehicles?

2.16 JOURNAL BEARING

The portion of a shaft, which revolves in the bearing and subjected to load at right angle to the axis of the shaft, is known as *journal* as clearly indicated in Figure 2.16. The whole unit consisting of the journal and its supporting part (or bearing) is known as *journal bearing*.

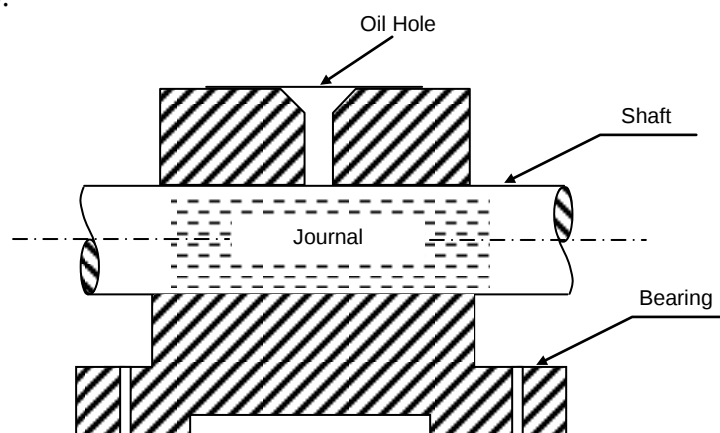


Figure 2.16 : Journal Bearing

2.17 ROLLING FRICTION

The frictional resistance arises only when there is relative motion between the two connecting surfaces. When there is no relative motion between the connecting surfaces or stated plainly when one surface does not slide over the other question of occurrence of frictional resistance or frictional force does not arise.

When a wheel rolls over a flat surface, there is a line contact between the two surfaces, parallel to the central axis of the cylinder. On the other hand when a spherical body rolls over a flat surface, there is a point contact between the two. In both the above mentioned cases there is no relative motion of slip between the line or point of contact on the flat surface because of the rolling motion. If while rolling of a wheel or that of a spherical body on the flat surface there is no deformation of depression of either of the two under the load, it is said to be a case pure rolling.

In practice it is not possible to have pure rolling and it can only be approached. How-so-ever hard the material be, either the rolling body will be deformed as happens in case of a car or cycle tyre, indicated in Figure 2.17(a) or the flat surface gets depressed or deformed. When the road roller passes over unsettled road or kacha road as shown in Figure 2.17(b). The surface is depressed.

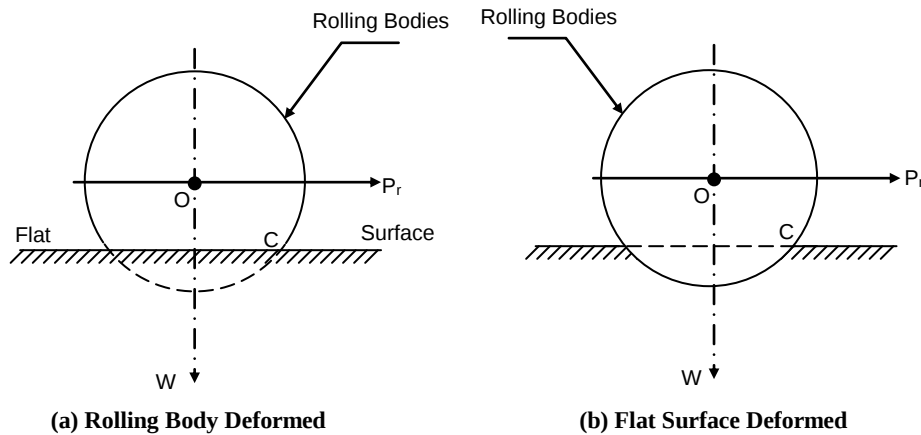


Figure 2.17 : Rolling Friction

At times when a load or a heavy machine or its part is to be shifted from one place to another place, for a short distance, and no suitable mechanical lifting device is available, the same is placed on a few rollers in the form of short pieces of circular bars or pipes as shown in Figure 2.18 and comparatively with a less force the load is moved. The rollers, roll and the one which becomes free at the rear side is again placed in the front of the load and so on. Because of the reduced friction, it requires less force.

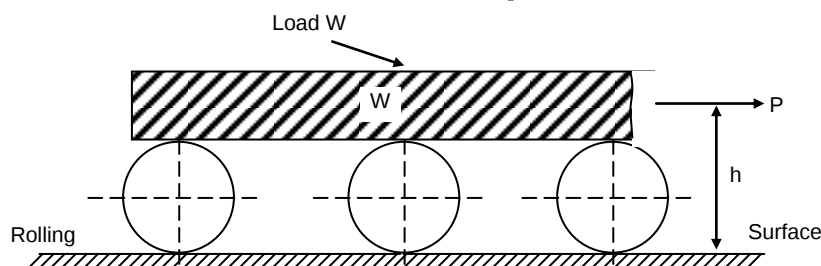


Figure 2.18 : Ball and Roller Bearing

When a shaft revolves in a bush bearing, there is sliding motion between the journal and the bearing surface, resulting in loss of power due to friction. If between the journal and the bearing surface, balls or rollers are provided, instead of sliding motion, rolling motion will take place. To reduce the coefficient of rolling friction the balls or rollers are made of chromium steel or chrome-nickel steel and they are further heat treated with a view to make them more hard. They are finally ground and polished with high precision.

Such an arrangement as mentioned above is provided in *Ball and Roller bearings*.

2.18 BALL AND ROLLER BEARINGS

Ball Bearing

A ball bearing is shown in Figure 2.19. As may be seen, it consists of mainly the following parts :

- (a) Inner ring,

- (b) Outer ring,
- (c) Anti-friction element in the form of balls placed between the two races, and
- (d) A cage which separates the balls from one another.

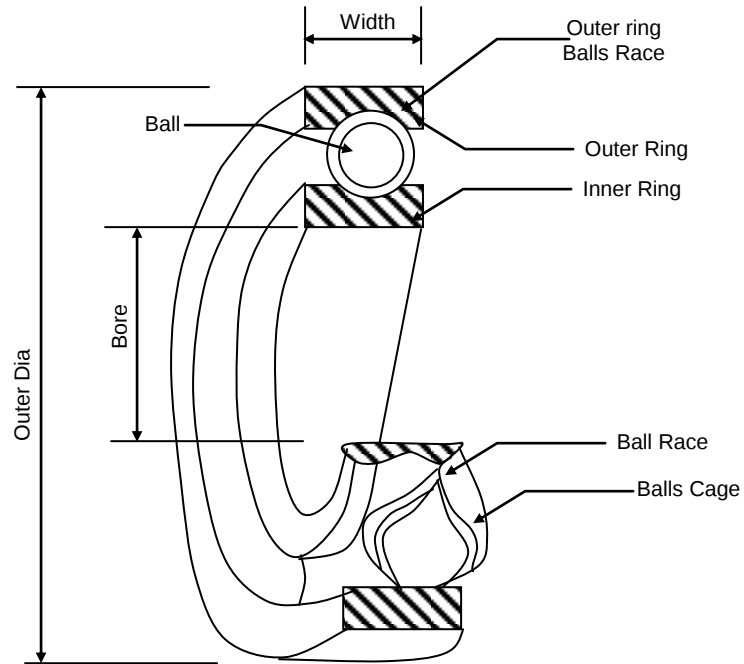


Figure 2.19 : Radial Ball Bearing

The inner ring is tight press-fitted to the shaft and the outer ring is press-fitted into a fixed housing which supports the whole bearing.

The cage is meant for keeping balls at fixed and equal distance from one another. Balls and races are made of best quality chromium steel containing high percentage of carbon with a view to reduce the value of coefficient of rolling friction, to the barest minimum, to make the bearing really an anti-friction.

Roller Bearing

A roller bearing is shown in Figure 2.20. There is no difference between ball bearing and the roller bearing except that in roller bearing rollers, instead of balls are used. In roller bearings, there is a line contact instead of point contact as in ball bearings. The roller bearings are widely used for more load carrying capacity than that of ball bearings.

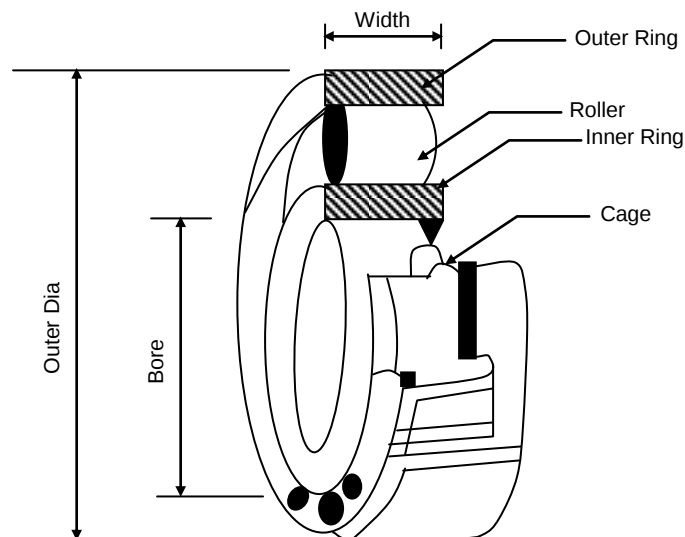


Figure 2.20 : Roller Bearing

The rollers may be cylindrical, straight or tapered. When tapered rollers are used, the bearings are called *tapered roller bearings* and those in which needles are used are called *needle bearings*.

Tapered Roller Bearings

Parts of a tapered roller bearing as well as its assembly are shown in Figure 2.21. The rollers are in the form of frustum of a cone. The contact angle is between 12° to 16° for thrusts of moderate magnitude and 28° to 30° for heavy thrust.

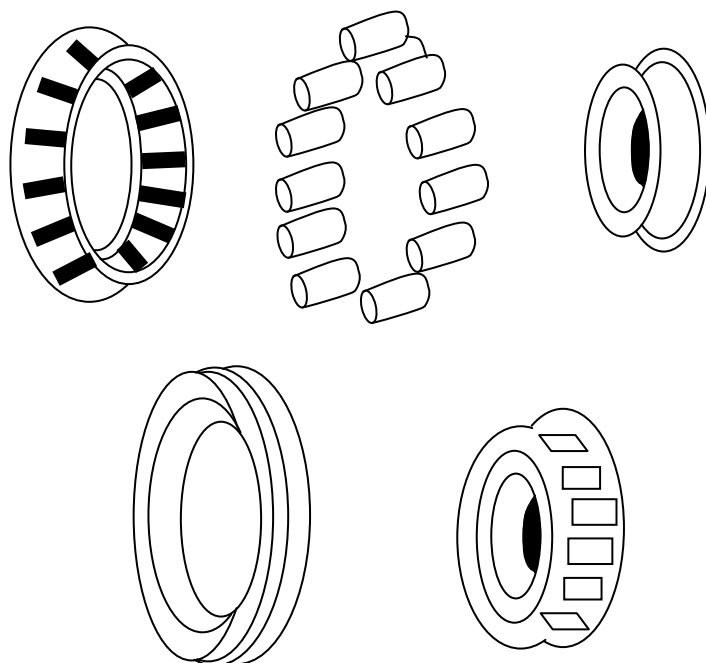


Figure 2.21 : Tapered Roller Bearing Parts

Needle Bearings

The rollers of small diameter are known as *needless*. It is : 2 to 4 mm. Ratio between length and diameter of needle is 3-10 : 1. Needle bearings are not provided with cage.

SAQ 6

Which type of bearing (journal bearing or ball bearing) has less friction?

2.19 SUMMARY

Friction force is generated when there is motion or tendency of motion of one body relative to other body in contact. It is useful in power transmission like belt drive, brakes and clutches. It is undesirable in bearings, etc. where it results in power loss.

Experiments were conducted to formulate laws of dry friction. According to the laws of dry friction, maximum force of friction depends on normal reaction and it is independent of area of contact and rubbing velocity. The ratio of the force of friction to the normal reaction is constant for the given surfaces and it is called coefficient of friction. The angle made by the resultant reaction to the normal reaction is called angle of friction.

If we unwind one helix of the screw it becomes an inclined plane. The theoretical expression of the screw jack has been derived by using this inclined plane. To make the screw self-locking, the helix angle should be less than the friction angle.

In some machines, shaft is subjected to the axial load. To support this load, pivot or collar bearing has to be used. When bearing is new, pressure distribution is uniform. Since wear is proportional to the product of pressure and radius, more wear takes place at larger radius. This results in non-uniform clearance between bearing and shaft and thereby reduction of pressure at larger radius and increase in pressure at the centre. Due to this, change, rate of wear becomes uniform. More power is lost in friction when pressure is uniform as compared to the case when ratio of wear is uniform.

To connect engine shaft with gear box, clutches are used. Two types of clutches are generally used in medium and heavy vehicle and they are – cone clutch and single plate clutch. In cars, buses and trucks, single plate clutch is used. Power transmitted by cone clutch is more than single plate clutch. Therefore, it is used where torque transmitted is more.

To support load at the supports, bearings are used. It can be journal bearing, ball bearing or roller bearing. In journal bearing there is a film of the lubricant between shaft and the bearing. The load is supported by the pressure generated in the layer of the lubricant. In case of ball and roller bearing, contact may be through a point or a line. The power loss due to friction will be very low in ball and roller bearing as compared to journal bearing.

2.20 KEY WORDS

Dry Friction	: It is the friction between two dry surfaces.
Lubricated Surfaces	: The lubricant which is put between surfaces forms a film between surfaces to reduce friction.
Static Friction	: It is the friction between the two static surfaces.
Kinetic Friction	: It is the friction between the surfaces having motion.
Coefficient of Friction	: It is the ratio of force of limiting friction to the normal reaction.
Limiting Friction	: It is the maximum force of friction which is developed when slide is impending.
Angle of Repose	: It is the angle of the inclined plane to the horizontal at which body slides on its own.
Screw Jack	: It is machine to lift or lower the load.
Pivots	: It is a surface which bears axial thrust of shaft.
Collars	: Collars are like plates which support axial thrust.
Clutch	: It is a friction coupling which is used between engine and gear box in an automobile.
Journal Bearing	: In these bearings, lubricant layer separate metal surfaces to lower the friction.
Ball Bearing	: In this bearing, balls are used to separate rotating surface from stationary surface.
Roller Bearing	: In this bearing rollers are used to separate rotating surface from stationary surface.

ANSWERS TO SAQs

Friction

Refer the relevant preceding text in the unit or other useful books on the topic listed in section 'Further Reading' to get the answer of the SAQs. These questions are helpful to prepare you for the examination.