Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

1

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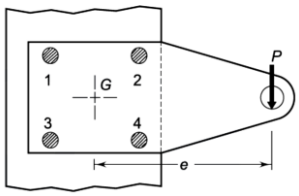
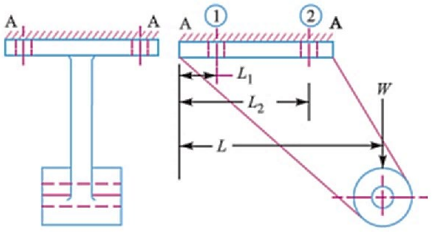
Eccentrically Loaded Bolted Joints

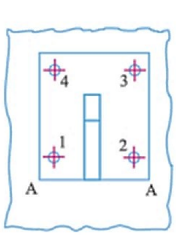
There are many applications of the bolted joints which are subjected to eccentric loading such as a wall bracket, pillar crane, etc.

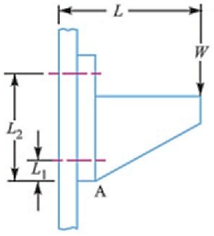
The eccentric load may be

1. In the plane containing the bolts.
2. Parallel to the axis of the bolts, and
3. Perpendicular to the axis of the bolts







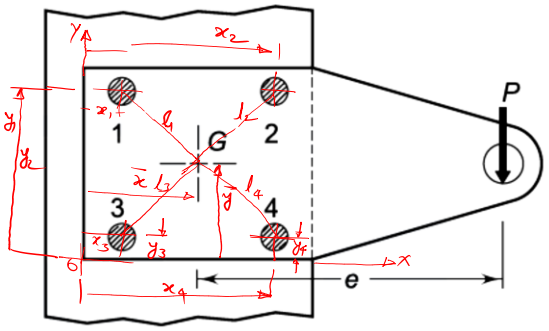


2

Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

1. In the plane containing the bolts.



To find the C.G of the bolted system.

Let A_1, A_2, A_3, A_4 be the area of cross sections of bolt 1, 2, 3, 4 respectively

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3 + A_4x_4}{A_1 + A_2 + A_3 + A_4}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} A_i x_i}{\sum A_i}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} A_i y_i}{\sum A_i}$$

Pxe

3

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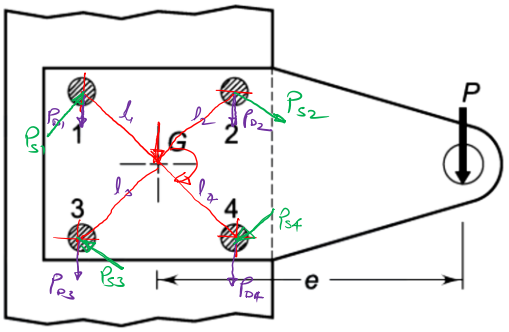
Eccentrically Loaded Bolted Joints

1. In the plane containing the bolts.



① Direct shear force

② Secondary shear force due to moment

$$P_D = \frac{P}{n} = \frac{\text{Total load}}{\text{No of bolts}}$$

$P_s \propto l$

$P_{s1} \propto l_1 \Rightarrow P_{s1} = cl_1$

$P_{s2} \propto l_2 \Rightarrow P_{s2} = cl_2$

$P_{s3} \propto l_3 \Rightarrow P_{s3} = cl_3$

$P_{s4} \propto l_4 \Rightarrow P_{s4} = cl_4$

$P_{s1} \times l_1 + P_{s2} l_2 + P_{s3} l_3 + P_{s4} l_4 = Pxe$

$cl_1 \times l_1 + cl_2 \cdot l_2 + cl_3 \cdot l_3 + cl_4 \cdot l_4 = Pxe$

$c[l_1^2 + l_2^2 + l_3^2 + l_4^2] = Pxe \Rightarrow$

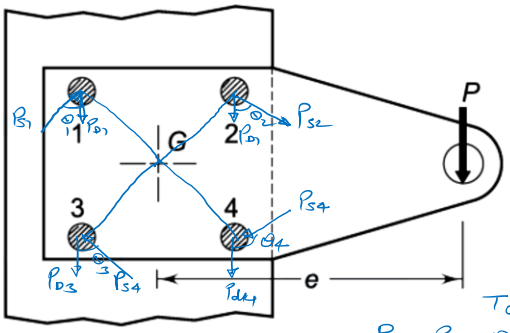
$$c = \frac{Pxe}{(l_1^2 + l_2^2 + l_3^2 + l_4^2)}$$

4

Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

1. In the plane containing the bolts.



$P_{s1} = cl_1 = \frac{Pel_1}{(l_1^2 + l_2^2 + l_3^2 + l_4^2)}$

$P_{s2} = cl_2 = \frac{Pel_2}{(l_1^2 + l_2^2 + l_3^2 + l_4^2)}$

$P_{s3} = cl_3 = \frac{Pel_3}{(l_1^2 + l_2^2 + l_3^2 + l_4^2)}$

$P_{s4} = cl_4 = \frac{Pel_4}{(l_1^2 + l_2^2 + l_3^2 + l_4^2)}$

Total load on each bolt P_1, P_2, P_3, P_4

$P_1 = \sqrt{P_{s1}^2 + P_{d1}^2 + 2P_{s1}P_{d1}\cos\theta_1}$, $P_3 = \sqrt{P_{s3}^2 + P_{d3}^2 + 2P_{s3}P_{d3}\cos\theta_3}$

$P_2 = \sqrt{P_{s2}^2 + P_{d2}^2 + 2P_{s2}P_{d2}\cos\theta_2}$, $P_4 = \sqrt{P_{s4}^2 + P_{d4}^2 + 2P_{s4}P_{d4}\cos\theta_4}$

$P_{max} = \max(P_1, P_2, P_3, P_4)$

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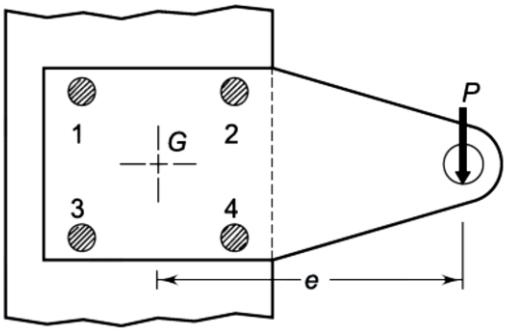
Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

1. In the plane containing the bolts.



$\tau = \frac{P_{max}}{A} = \frac{P_{max}}{\frac{\pi}{4}d^2}$

calculate d

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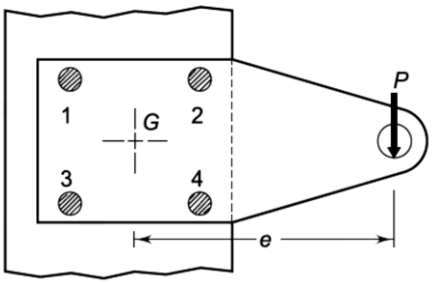
Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

Problem:

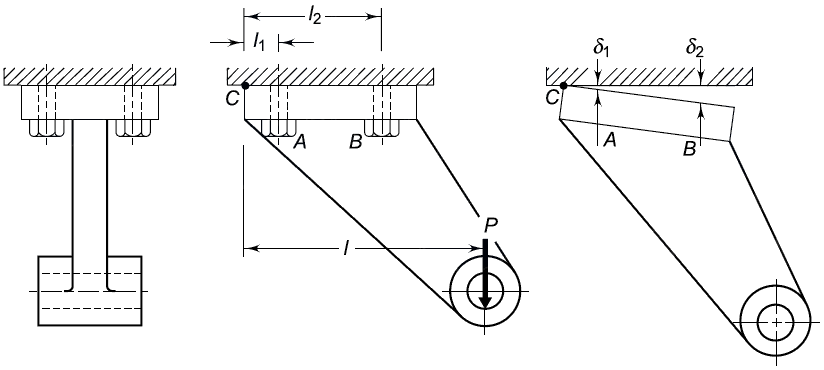
The structural connection shown in Fig. is subjected to an eccentric force P of 10 kN with an eccentricity of 500 mm from the CG of the bolts. The centre distance between bolts 1 and 2 is 200 mm, and the centre distance between bolts 1 and 3 is 150 mm. All the bolts are identical. The bolts are made from plain carbon steel 30C8 (S_{yt} = 400 N/mm^2) and the factor of safety is 2.5. Determine the size of the bolts.



Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

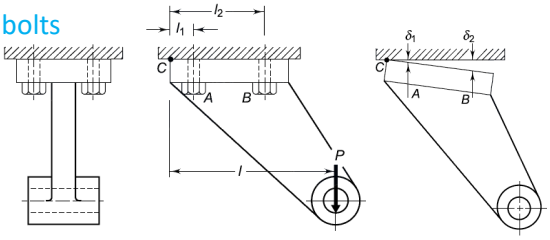
2. Parallel to the axis of the bolts



Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

2. Parallel to the axis of the bolts



The bolts are subjected to following stresses:

(i) Direct tensile stress due to load *P*.

(ii) Tensile stress due to tendency of the bracket to tilt in clockwise direction about the edge *c*.

Step I Direct tensile force

Since the bolts are identical, the direct tensile force on each bolt is given by,

$$P_{d1} = P_{d2} = \frac{P}{(No.\,of\ Bolts)}$$

Step II Tensile force due to tendency of bracket to tilt

The following assumptions are made:

(i) All bolts are identical.

(ii) The bracket and the structure are rigid.

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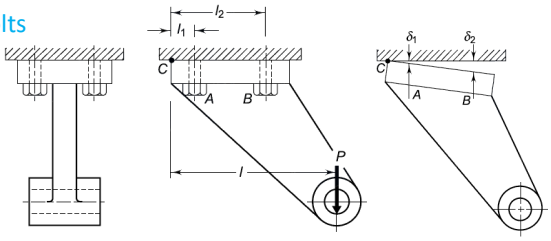
Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

2. Parallel to the axis of the bolts



Step II Tensile force due to tendency of bracket to tilt

(iii) The bolts are not preloaded and there is no initial tensile stress due to tightening of the bolt.

(iv) As shown in Fig., when the load tends to tilt the bracket about the edge c, each bolt is stretched by an amount (δ), which is proportional to its distance from the tilting edge. Or, $\delta_1 \propto l_1$ $\delta_2 \propto l_2$

Also,

Force ∝ Stress because (P=σA),

Stress ∝ Strain because (σ=Eε)

Strain ∝ Strech because (ε= δ/l)

Therefore, it can be concluded that the resisting force acting on the bolt due to the tendency of bracket to tilt, is proportional to its distance from the tilting edge. The bolts at *A* are denoted by 1, 3 and bolts at *B* by 2, 4.

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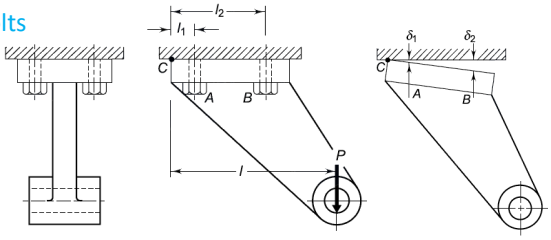
Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

2. Parallel to the axis of the bolts



Suppose C is the load in the bolt per unit distance from the tilting edge, due to the tilting effect of the bracket. Then forces acting on the bolts are given by,

$f_1 = Cl_1$, $f_2 = Cl_2$

Equating the moments of these resisting forces about the tilting edge to the moment due to the external force,

$Pl = 2f_1l_1 + 2f_2l_2 = 2(Cl_1)l_1 + 2(Cl_2)l_2 = 2C(l_1^2+l_2^2)$ $\therefore \quad C = \frac{Pl}{2(l_1^2+l_2^2)}$ {Total four bolts

$f_1 = \frac{Pll_1}{2(l_1^2+l_2^2)}$, $f_2 = \frac{Pll_2}{2(l_1^2+l_2^2)}$, $f_3 = \frac{Pll_3}{2(l_1^2+l_2^2)}$ $f_4 = \frac{Pll_4}{2(l_1^2+l_2^2)}$

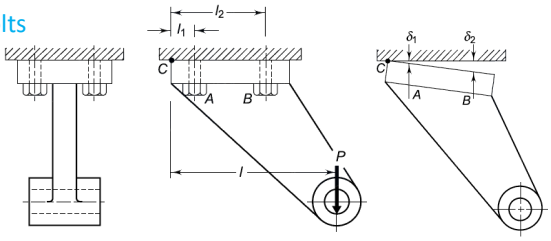
$l_3 = l_1$, $l_4 = l_2$

Total load on Bolt 1 $P_1 = P_{d1} + f_1$, $P_2 = P_{d2} + f_2$

Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

2. Parallel to the axis of the bolts



$P_3 = P_{d3} + f_3$

$P_4 = P_{d4} + f_4$

$P_{max} = \max(P_1, P_2, P_3, P_4)$

$\sigma_t = \frac{P_{max}}{A} = \frac{P_{max}}{\frac{\pi}{4}d_c^2}$

Calculate d_c & specify size of the bolt from table of standard sizes of bolt

Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

Problem: 2 A cast iron bracket, supporting the transmission shaft and the belt pulley, is fixed to the steel structure by means of four bolts as shown in Fig. There are two bolts at A and two bolts at B. The tensions in slack and tight sides of the belt are 5 kN and 10 kN respectively. The belt tensions act in a vertically downward direction. The distances are as follows, l_1= 50 mm l_2 = 150 mm l = 200 mm.

The maximum permissible tensile stress in any bolt is 60 N/mm^2. Determine the size of the bolts.

$P_{d_1} = P_{d_2} = \frac{P}{4} = \frac{15}{4} = 3.75$kN

$f_1 = cl_1$, $f_2 = cl_2$

$P \times l = (f_1 l_1 + f_2 l_2) \times 2 = 2c(l_1^2 + l_2^2)$

$c = \frac{Pl}{2(l_1^2 + l_2^2)} = \frac{15000 \times 200}{2(50^2 + 150^2)}$

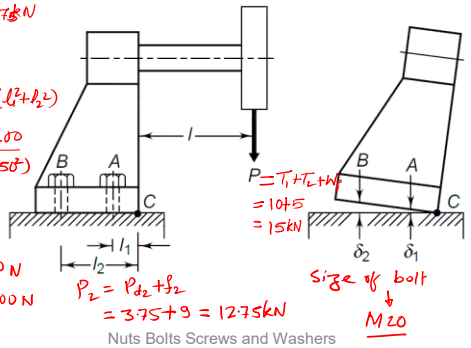
$= 60$ N/mm

$f_1 = cl_1 = 60 \times 50 = 3000$ N

$f_2 = cl_2 = 60 \times 150 = 9000$ N

$P_1 = P_{d_1} + f_1 = 3.75 \times 1000 + 3000$ N

$= 6.75$kN



$P_2 = P_{d_2} + f_2$

$= 3.75 + 9 = 12.75$kN

$P_{max} = 12.75$kN

$\sigma_{t max} = \frac{P_{max}}{\frac{\pi}{4} d_c^2}$

$\frac{\pi}{4} d_c^2 = \frac{P_{max}}{\sigma_{t max}}$

$= \frac{12.75 \times 1000}{60}$

$= 212.5$ mm^2

$d_c = \sqrt{\frac{212.5 \times 4}{\pi}}$

$d_c = 16.44$ mm

Size of bolt

M20

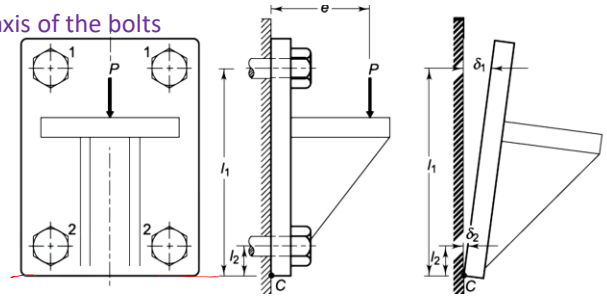
Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

3. Perpendicular to the axis of the bolts

A bracket, fixed to the steel structure by means of four bolts, is shown in Fig.

It is subjected to eccentric force *P* at a distance *e* from the structure. The force *P* is perpendicular to the axis of each bolt.



The lower two bolts are denoted by 2, while the upper two bolts by 1. In this analysis, the following assumptions are made:

(i) The bracket and the steel structure are rigid.

(ii) The bolts are fitted in reamed and ground holes.

(iii) The bolts are not preloaded and there are no tensile stresses due to initial tightening.

Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

3. Perpendicular to the axis of the bolts

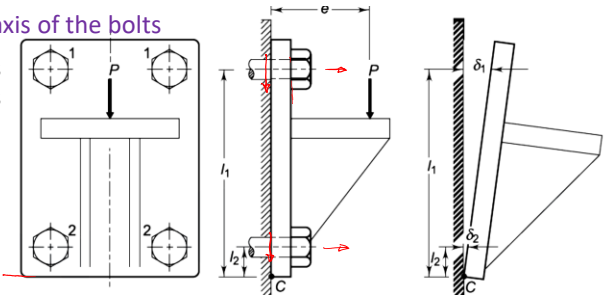
(iv) The stress concentration in threads is neglected.

(v) All bolts are identical.

The bolts are subjected to following stresses:

(i) Direct shear stress due to load *P*.

(ii) Tensile stress due to tendency of the bracket to tilt in clockwise direction about the edge *c*.



Since the bolts are identical, the direct shear force on each bolt is given by, $P_{s1} = P_{s2} = \frac{P}{(No.\ of\ Bolts)}$

As shown in Fig., when the load tends to tilt the bracket about the edge c, each bolt is stretched by an amount (δ), which is proportional to its distance from the tilting edge.

Or, $\delta_1 \propto l_1$ $\delta_2 \propto l_2$

Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

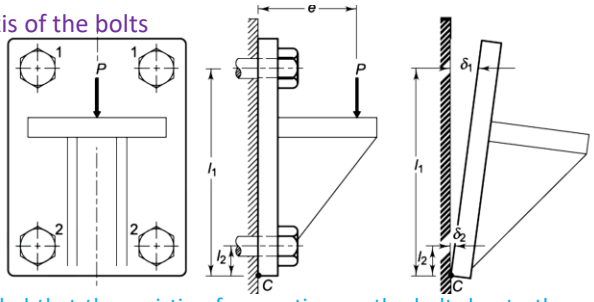
3. Perpendicular to the axis of the bolts

Also,

Force ∝ Stress because (P=σA),

Stress ∝ Strain because (σ=Eε)

Strain ∝ Strech because (ε= δ/l)



Therefore, it can be concluded that the resisting force acting on the bolt due to the tendency of bracket to tilt, is proportional to its distance from the tilting edge.

Suppose *C* is the load in the bolt per unit distance from the tilting edge, due to the tilting effect of the bracket. Then tensile forces acting on the bolts are given by,

$f_1 = Cl_1$, $f_2 = Cl_2$

Equating the moments of these resisting forces about the tilting edge to the moment due to the external force,

$Pl = 2f_1l_1 + 2f_2l_2 = 2(Cl_1)l_1 + 2(Cl_2)l_2 = 2C(l_1^2+l_2^2)$

Stresses in Screw Fasteners

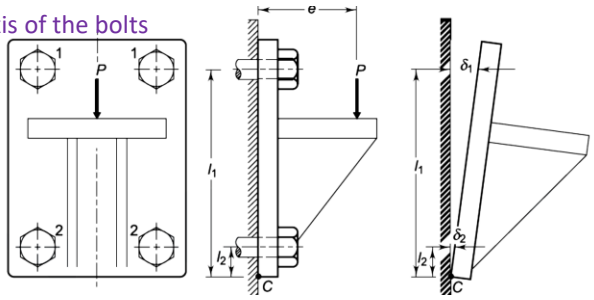
Eccentrically Loaded Bolted Joints

3. Perpendicular to the axis of the bolts

$c = \frac{Pe}{2(l_1^2 + l_2^2)}$

$f_1 = cl_1 = \frac{Pel_1}{2(l_1^2 + l_2^2)}$

$f_2 = cl_2 = \frac{Pel_2}{2(l_1^2 + l_2^2)}$



$f_{max} = f_1$ (f_1 is situated at the farthest distance from tilting edge c)

Therefore the tensile stress is $\sigma_t = \frac{f_{max}}{\frac{\pi}{4} d_c^2}$

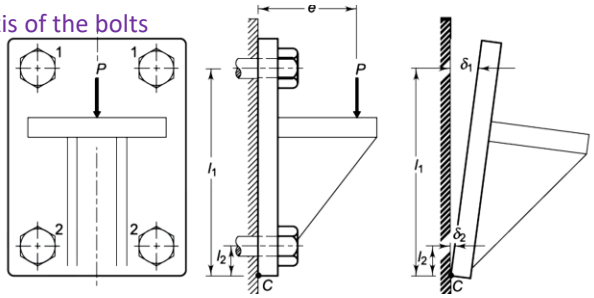
& direct shear stress $\tau = \frac{P_{s1}}{A}$

Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

3. Perpendicular to the axis of the bolts

$\tau = \frac{P_{s1}}{\frac{\pi}{4} d^2}$



The bolts can be designed on the basis of principal stress theory or principal shear stress theory.

The principal stress σ_1 is given by, $\sigma_1 = \frac{\sigma_t}{2} + \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$

Using these relations, bolt size may be determined.

The principal shear stress is given by, $\tau_{max} = \sqrt{\left(\frac{\sigma_t}{2}\right)^2 + \tau^2}$

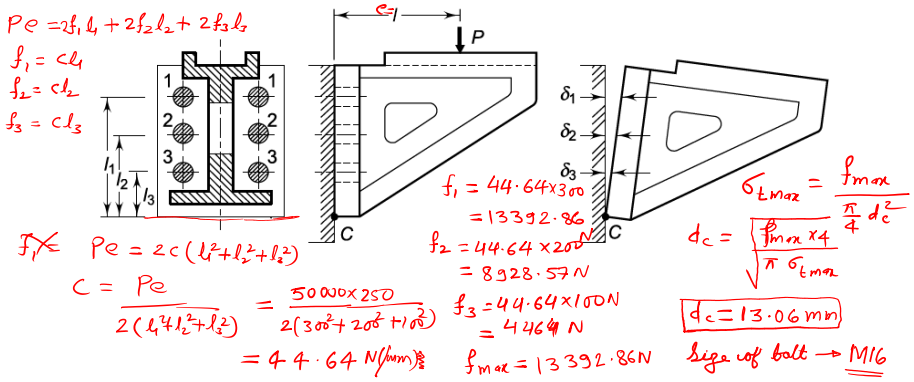
The bolt material is usually ductile.

Therefore, it is appropriate to use max shear stress theory.

Stresses in Screw Fasteners

Eccentrically Loaded Bolted Joints

Problem: 3 *A bracket is fastened to the steel structure by means of six identical bolts as shown in Fig. Assume the following data: l_1 = 300 mm l_2 = 200 mm l_3 = 100 mm l = 250 mm P = 50 kN Neglecting shear stress, determine the size of the bolts, if the maximum permissible tensile stress in any bolt is limited to 100 N/mm^2.*



$Pe = 2f_1 l_1 + 2f_2 l_2 + 2f_3 l_3$

$f_1 = cl_1$

$f_2 = cl_2$

$f_3 = cl_3$

$f_1 = 44.64 \times 300 = 13392.86$ N

$f_2 = 44.64 \times 200 = 8928.57$ N

$f_3 = 44.64 \times 100 = 4464$ N

$f_{max} = 13392.86$ N

$\sigma_{t\,max} = \frac{f_{max}}{\frac{\pi}{4} d_c^2}$

$d_c = \sqrt{\frac{f_{max} \times 4}{\pi\, \sigma_{t\,max}}}$

$d_c = 13.06$ mm

Size of bolt → M16

$P e = 2c(l_1^2 + l_2^2 + l_3^2)$

$c = \frac{Pe}{2(l_1^2 + l_2^2 + l_3^2)} = \frac{50000 \times 250}{2(300^2 + 200^2 + 100^2)} = 44.64$ N/mm