Lecture # 17-18

Design of Shafts

Design of Shafts

Introduction

Shaft is a common and important rotating machine element supporting various transmission members such as pulleys, gears, etc mounted by means of keys or splines, to transmit power from one place to another.

The cross section of the shaft is circular and it may be hollow or solid.

The shaft is supported on bearings.

The shaft is generally acted upon by bending moment, torsion and axial force.

The design procedure of shaft include determining the stresses at the point of highest loading section (critical point).

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There are two common terms which are similar to shaft and used are axle and spindle.

Axle is a non-rotating member used for supporting rotating wheels etc. and do not transmit any torque.

Spindle is simply defined as a short shaft.

However, design method remains the same for axle and spindle as that for a shaft.

Standard sizes of Shafts

Typical sizes of solid shaft that are available in the market are,

Up to 25 mm 0.5 mm increments

25 to 50 mm 1.0 mm increments

50 to 100 mm 2.0 mm increments

100 to 200 mm 5.0 mm increments

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Material for Shafts

The material used for shafts should have high strength, good machinability, low notch sensitivity factor, good heat treatment and high wear resistant properties

The ferrous, non-ferrous materials and non metals are used as shaft material depending on the application.

Some of the common ferrous materials used for shaft are

Hot-rolled plain carbon steel

These materials are least expensive.

Since it is hot rolled, scaling is always present on the surface and machining is required to make the surface smooth.

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Cold-drawn plain carbon/alloy composition

Since it is cold drawn it has got its inherent characteristics of smooth bright finish.

Amount of machining therefore is minimal.

Better yield strength is also obtained.

This is widely used for general purpose transmission shaft.

Alloy steels

Alloy steel as one can understand is a mixture of various elements with the parent steel to improve certain physical properties.

To retain the total advantage of alloying materials one requires heat treatment of the machine components after it has been manufactured.

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Nickel, chromium and vanadium are some of the common alloying materials.

However, alloy steel is expensive.

These materials are used for relatively severe service conditions.

When the situation demands great strength then alloy steels are used.

They have fewer tendencies to crack, warp or distort in heat treatment.

Residual stresses are also less compared to CS(Carbon Steel).

In certain cases the shaft needs to be wear resistant, and then more attention has to be paid to make the surface of the shaft to be wear resistant.

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The common types of surface hardening methods are,

Hardening of surface

Case hardening and carburizing

Cyaniding and nitriding

Manufacturing of Shafts

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding.

The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses.

The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut.

Shafts of larger diameter are usually forged and turned to size in a lathe.

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Stresses in Shafts

The following stresses are induced in the shafts :

1. Shear stresses due to the transmission of torque (i.e. due to torsional load).

2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements

like gears, pulleys etc. as well as due to the weight of the shaft itself.

3. Stresses due to combined torsional and bending loads.

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Design considerations for shaft

For the design of shaft following two methods are adopted,

Design based on Strength

In this method, design is carried out so that stress at any location of the shaft should not exceed the material yield stress.

However, no consideration for shaft deflection and shaft twist is included.

Design based on Stiffness

Basic idea of design in such case depends on the allowable deflection and twist of the shaft.

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Design considerations for shaft

In designing shafts on the basis of strength, the following cases may be considered :

(a) Shafts subjected to twisting moment or torque only,

(b) Shafts subjected to bending moment only,

(c) Shafts subjected to combined twisting and bending moments, and

(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

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Design of Shafts on the basis of Strength

Basic stress equations :

$$\text{Bending Stress } \sigma_b = \frac{32M}{\pi d_o^{\,3}\left(1-k^4\right)}$$

Where,

M : Bending moment at the point of interest

d_o : Outer diameter of the shaft

k : Ratio of inner to outer diameters of the shaft

(k = 0 for a solid shaft because inner diameter is zero)

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Design of Shafts on the basis of Strength

$$\text{Axial Stress } \sigma_a = \frac{4\alpha F}{\pi d_o^3 \left(1 - k^2\right)}$$

Where,

F: Axial force (tensile or compressive)

α: Column-action factor = 1.0 (for tensile load)

The term α is known as column action factor used to take care the phenomenon of buckling of long slender members which are acted upon by axial compressive loads.

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Design of Shafts on the basis of Strength

Here, α is defined as,

$$\alpha = \frac{1}{1 - 0.0044 \, (L/K)} \quad \text{for} \quad \frac{L}{K} < 115$$

$$\alpha = \frac{\sigma_{yc}}{\pi^2 nE} \left(\frac{L}{K} \right)^2 \quad \text{for} \quad \frac{L}{K} > 115$$

Where,

n = 1.0 for hinged end

n = 2.25 for fixed end

n = 1.6 for ends partly restrained, as in bearing

K = least radius of gyration,

L = shaft length

σ_{yc}= yield stress in compression

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Design of Shafts on the basis of Strength

$$\text{Stress due to torsion } \tau_{xy} = \frac{16T}{\pi d_o^3\left(1-k^4\right)}$$

Where,

T : Torque on the shaft

τ_{xy} : Shear stress due to torsion

Combined Bending and Axial stress

Both bending and axial stresses are normal stresses, hence the net normal stress is given by,

The net normal stress can be either positive or negative.

$$\sigma_x = \left[\frac{32M}{\pi d_o^3\left(1-k^4\right)} \pm \frac{4\alpha F}{\pi d_o^3\left(1-k^2\right)}\right]$$

Normally, shear stress due to torsion is only considered in a shaft and shear stress due to load on the shaft is neglected.

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Shafts Subjected to Combined Twisting Moment and Bending Moment

The following two theories are important from the subject point of view :

1.Maximum shear stress theory or Guest's theory.

2.It is used for ductile materials such as mild steel.

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4)$$

2. Maximum normal stress theory or Rankine’s theory.

It is used for brittle materials such as cast iron.

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

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ASME design Code

The shafts are normally acted upon by gradual and sudden loads. Hence, the above equations is modified in ASME code by suitable load factors,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$M_e = \frac{1}{2}\left((K_m \times M) + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right)$$

where

K_m = Combined shock and fatigue factor for bending, and

K_t = Combined shock and fatigue factor for torsion.

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The following table shows the recommended values for K_m and K_t.

Nature of Load	K_m	K_t
1. Stationary Shafts		
(a) Gradually Applied Load	1.0	1.o
(b) Suddenly Applied Load	1.5 to 2.0	1.5 to 2.0
2. Rotating Shafts		
(a) Gradually Applied Load or Steady Load	1.5	1.0
(b) Suddenly Applied Load with minor shocks only	1.5 to 2.0	1.5 to 2.0
(c) Suddenly Applied Load with heavy shocks	2.0 to 3.0	1.5 to 3.0

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Design of Shafts on the basis of Rigidity

Sometimes the shafts are to be designed on the basis of rigidity.

Following are the two types of rigidity.

1. Torsional rigidity

The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected.

The permissible amount of twist should not exceed 0.25° per metre length of such shafts.

For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value.

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1. Torsional rigidity

The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft.

The torsional deflection may be obtained by using the torsion equation, $\frac{T}{J} = \frac{G \cdot \theta}{L}$ or $\theta = \frac{T \cdot L}{J \cdot G}$ where

θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

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2. Lateral rigidity.

It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces.

The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment.

If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials.

But when the shaft is of variable cross-section, then the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *i.e.*

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

Design of Shafts

Example 1

A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

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Example 2

A shaft supported at the ends in ball bearings carries a straight tooth spur gear at its mid span and is to transmit 7.5 kW at 300 r.p.m. The pitch circle diameter of the gear is 150 mm. The distances between the centre line of bearings and gear are 100 mm each. If the shaft is made of steel and the allowable shear stress is 45 MPa, determine the diameter of the shaft. Show in a sketch how the gear will be mounted on the shaft; also indicate the ends where the bearings will be mounted? The pressure angle of the gear may be taken as 20°.

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Example 3

A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 metres. It carries two pulleys each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.

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Example 4

A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is 1.5 metre in diameter and has belt tensions 5.4 kN and 1.8 kN on the tight side and slack side of the belt respectively. Both these tensions may be assumed to be vertical. If the pulley be overhang from the shaft, the distance of the centre line of the pulley from the centre line of the bearing being 400 mm, find the diameter of the shaft. Assuming maximum allowable shear stress of 42 MPa.

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Example 5

A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and $\mu = 0.24$. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.

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Example 6

A shaft is supported on bearings A and B, 800 mm between centres. A 20° straight tooth spur gear having 600 mm pitch diameter, is located 200 mm to the right of the left hand bearing A, and a 700 mm diameter pulley is mounted 250 mm towards the left of bearing B. The gear is driven by a pinion with a downward tangential force while the pulley drives a horizontal belt having 180° angle of wrap. The pulley also serves as a flywheel and weighs 2000 N. The maximum belt tension is 3000 N and the tension ratio is 3 : 1. Determine the maximum bending moment and the necessary shaft diameter if the allowable shear stress of the material is 40 MPa.

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Example 7

A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.

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Example 8

Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.

References

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